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On Helmholtz resonators with tapered necks

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Abstract

The acoustical properties of Helmholtz resonators with necks having cross-section dimensions decreasing away from the entry of the resonator cavities are investigated experimentally and theoretically in the present study. Results show that significant improvement of the sound absorption capacity of the resonators can be obtained by introducing such neck tapering. Such improvement is enhanced when the tapered length is increased. The present derived theory predicts the resonance frequency of the tapered neck resonator within engineering tolerance and the accuracy is again improved as the tapered length increases.

1. Introduction

Helmholtz resonator is widely used as a silencing device in the ducted system because of its strong sound attenuation property once appropriately tuned (for instance, Ref. [1]). It has been very useful in dealing with the low-frequency noise propagating inside the air conditioning ductworks in which the flow velocities are usually less than 20 m/s [2]. Good design of such resonator is of great importance in building noise control as the conventional dissipative type silencers are usually not effective for low-frequency noise attenuation and they also produce significant static air pressure loss. Though the development of active control has provided another means to tackle the low-frequency duct noise problem [3], the stable as well as robust performance of the Helmholtz resonator makes it one of the best choices in the noise control industry.

The importance of the Helmholtz resonator has attracted the attention of many researchers and engineers. In the past few decades, efforts have been made extensively on how to predict and improve the transmission loss produced by such resonator (for instance, Refs. [4,5]). Many different geometries and complicated arrangements have been considered. For instance, Chanaud

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[6] studied the effects of different orifice shapes, Dicky and Selamet [7] investigated the cavity aspect ratio effects, Selamet and Lee [8] proposed extending the neck into the resonator cavity and Griffin et al. [9] suggested coupling two resonators. The idea of coupling Helmholtz resonator with dissipative silencers has also been proposed [10]. Though this list is by no mean exhaustive, some of these suggestions appear bulky or are less easily manufactured. The aeroacoustic effect of the Helmholtz resonator is also studied [11].

The acoustic impedance at the outlet of the Helmholtz resonator affects significantly the achievable sound absorption coefficient if it is used as wall lining and sound transmission loss if it is applied to a duct. The highest absorption or sound transmission loss takes place near to the resonance frequency and the extent of the effect therefore depends on the resistive part of the acoustic impedance. The low-frequency plane wave theory suggests that a reduction of this resistance gives rise to higher sound transmission loss, but will only increase the sound absorption coefficient if this resistance is higher than that of the medium, which is air in building noise control. The proof for this conclusion can be obtained from standard textbooks, for instance Kinsler et al. [12], and thus is not presented here. Since a smoother area change from the neck towards the cavity will reduce the flow resistance, tapering the neck appears to be a way to enhance the performance of a Helmholtz resonator without significant alteration of geometry. However, the resonance frequency may be affected by the tapered neck.

In the present study, the acoustic impedance and the sound absorption coefficient of a Helmholtz resonator with a tapered neck is investigated experimentally. A simplified theory is derived in an attempt to explain the experimental results and to predict the corresponding resonance frequency. It is hoped that the present results can contribute to the future design of the Helmholtz resonator.

2. Simplified theory of tapered neck resonator

Conventional Helmholtz resonator consists of a cavity connected to a neck having length and cross-sectional diameter much smaller than the dimension of the cavity [12]. Fig. 1 shows the schematic of the tapered neck resonator and the nomenclatures adopted in the present study. The resonator considered in the present study has a cylindrical cavity of volume V and a neck with tapered section of length L, inlet radius r_i and outlet radius r_o. A short parallel section of length L_p is allowed inside the neck before the tapered section. The wavelength of the sound considered is assumed to be much longer than the neck dimension.

A simplified situation is that the sound wave propagating inside the tapered neck remains planar so that the wave equation with the tapered neck boundaries can approximately be analytically solved. Suppose p represents the sound pressure in the neck, the plane wave assumption suggests [12]

$$\frac{1}{A}\frac{\mathrm{d}}{\mathrm{d}x}\left(A\frac{\mathrm{d}p}{\mathrm{d}x}\right)+k^2p=0, \tag{1}$$

where k denotes the wavenumber and A the cross-section area of the neck at the axial distance x which can be written as $A=\pi(mx+r_i)^2$, where m is the slope of the tapered neck and $m=$

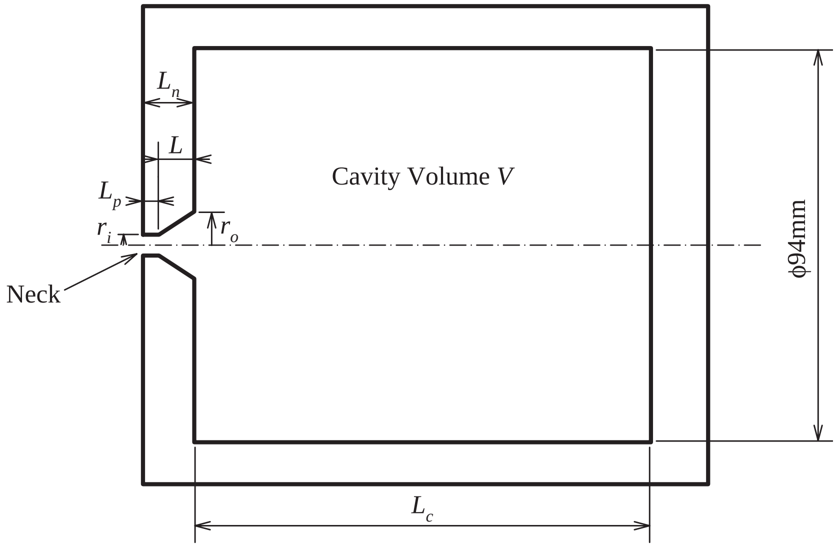


Fig. 1. Schematic of the tapered neck Helmholtz resonator and the nomenclature adopted.

$(r_o - r_i)/L$. Eq. (1) leads to the following ordinary differential equation for p:

$$\frac{\mathrm{d}^2 p}{\mathrm{d}x^2} + \frac{2m}{mx + r_i}\frac{\mathrm{d}p}{\mathrm{d}x} + k^2 p = 0. \tag{2}$$

Substituting $\alpha = mx + r_i$ into Eq. (2), one obtains

$$\frac{\mathrm{d}^2 p}{\mathrm{d}\alpha^2} + \frac{2}{\alpha}\frac{\mathrm{d}p}{\mathrm{d}x} + \left(\frac{k}{m}\right)^2 p = 0 \tag{3}$$

whose solutions can be readily found by using standard series method (for instance, see Ref. [13]):

$$p = \frac{m}{k\alpha}(c_1 \mathrm{e}^{-\mathrm{i}k\alpha/m} + c_2 \mathrm{e}^{\mathrm{i}k\alpha/m}) = \frac{m}{k(mx + r_i)}(c_1 \mathrm{e}^{-\mathrm{i}k(mx+r_i)/m} + c_2 \mathrm{e}^{\mathrm{i}k(mx+r_i)/m}), \tag{4}$$

where c_1 and c_2 are complex constants depending on the boundary conditions at $x = 0$ and L. The location $x = L$ is the inlet to the resonator cavity. The acoustic impedance there z_o is

$$z_o = \left.\frac{p}{\rho c u}\right|_{x=L}, \tag{5}$$

where ρ is the mean air density, c the ambient speed of sound and u the acoustic particle velocity which is given by the expression $u = (\mathrm{i}ck)^{-1}\mathrm{d}p/\mathrm{d}x$ according to the plane wave assumption. One can then find that

$$\frac{c_2}{c_1} = \frac{z_o(kr_o - \mathrm{i}m) - kr_o}{z_o(kr_o + \mathrm{i}m) + kr_o}\mathrm{e}^{-2\mathrm{i}kr_o/m}. \tag{6}$$

The acoustic impedance at the inlet of the tapered neck z_i is

$$z_i = \left.\frac{p}{\rho c u}\right|_{x=0} = kr_i \frac{c_1 \mathrm{e}^{-\mathrm{i}kr_i/m} + c_2 \mathrm{e}^{\mathrm{i}kr_i/m}}{(kr_i - \mathrm{i}m)c_1 \mathrm{e}^{-\mathrm{i}kr_i/m} - (kr_i + \mathrm{i}m)c_2 \mathrm{e}^{\mathrm{i}kr_i/m}}. \tag{7}$$

Combining Eqs. (6) and (7), one obtains

$$z_i = \frac{k^2 r_i r_o z_o + (\mathrm{i}k^2 r_i r_o - kr_i m z_o)\tan(kL)}{k^2 r_i r_o - \mathrm{i}km^2 L z_o + (kr_o m + \mathrm{i}m^2 z_o + \mathrm{i}k^2 r_i r_o z_o)\tan(kL)}. \tag{8}$$

For a cylindrical neck, $m = 0$, Eq. (8) converges to the well-known solution in existing literature (for instance, Ref. [14]). Since $kL \to 0$ within the functioning frequency range of a Helmholtz resonator,

$$z_i = \frac{r_i^2 z_o + \mathrm{i}kr_i r_o L}{r_o^2 + \mathrm{i}kr_i r_o L z_o}. \tag{9}$$

The pressure change inside the resonator cavity induced by a small movement δ of the wave into the cavity is

$$p = \frac{\rho c^2 A_{x=L}\delta}{V} = \frac{\rho c \pi r_o^2 u}{\mathrm{i}kV}, \tag{10}$$

where V is the cavity volume. If one excludes temporarily the energy dissipation at $x = L$ which is resistive to the wave motion (damping), z_o, is thus

$$z_o = \left.\frac{p}{\rho c u}\right|_{x=L} = \frac{\pi r_o^2}{\mathrm{i}kV}. \tag{11}$$

Therefore,

$$z_i = \frac{r_i^2\, \pi r_o^2/(\mathrm{i}kV) + \mathrm{i}kr_i r_o L}{r_o^2 + r_i r_o L\, \pi r_o^2/V} = \mathrm{i}\frac{r_i}{r_o}\frac{kL - \pi r_i r_o/(kV)}{1 + \pi r_i r_o\, L/V}. \tag{12}$$

The corresponding resonance frequency f_{res} is now

$$f_{res} = \frac{c}{2\pi}\sqrt{\frac{\pi r_i r_o}{LV}} = \frac{c}{2\pi}\sqrt{\frac{\pi r_i^2}{LV} + \frac{m\pi}{V}}, \tag{13}$$

suggesting an increase of the resonance frequency when the neck section running into the cavity is tapered. The actual increase decreases with increasing cavity volume V, but with increasing tapered section slope m.

If now one includes the resistance in z_o into consideration and let it be R_o, the acoustic impedance of the fully tapered neck Helmholtz resonator, $z_{i,res}$, becomes

$$z_{i,res} = \frac{r_i^2 R_o}{r_o^2 + r_i r_o L \pi r_o^2/V + \mathrm{i} r_i r_o R_o \sqrt{\pi r_i r_o L/V}}. \tag{14}$$

Though it is not easy to find exactly the resistance R_o, one expects it increases relatively quickly with frequency and thus k in general as the pressure loss for incompressible flow though enlargement suggests that R_o is proportional to the squared velocity [15]. Since the wavelength concerned is much longer than any length dimension of the resonator and R_o is finite, one expects

$$z_{i,res} \approx \frac{r_i^2}{r_o^2} R_o, \tag{15}$$

showing that this resistance is likely to be reduced by the tapered neck. One should note that the additional resistance due to radiation out of the neck inlet is unchanged under the present simplified theory. The current deduction tends to suggest a considerable improvement of the Helmholtz resonator's sound absorption coefficient at the resonance frequency is likely by tapering the neck.

Though kL_p is again very small and will not affect the main conclusions of the simplified theory, one can still anticipate this parallel section in front of the tapered neck section introduces additional inductance and will have some minor effects on the results of Eqs. (14) and (15). The total acoustic impedance after taking into account the effects of L_p, z_t, can be approximated as

$$z_t = \frac{z_i + \mathrm{i}\tan(kL_p)}{1 + \mathrm{i}z_i \tan(kL_p)} \approx \frac{z_i + \mathrm{i}kL_p}{1 + \mathrm{i}kL_p z_i} \quad \Rightarrow \quad |z_t|^2 \approx |z_i|^2 + (1 - |z_i|^4)(kL_p)^2. \tag{16}$$

As it will be shown later in the experiment that the magnitude of z_i at resonance in the present study is always higher than that of air, it can be deduced from Eq. (16) that the magnitude of z_t is even slightly smaller than $z_{i,res}$ at low frequency if L is fixed. A reduction in L_p with the total neck plate thickness L_n $(= L + L_p)$ and the tapered section slope (m) fixed further reduces this magnitude, but such reduction is very limited. The change of f_{res} with L_p is not obvious as the resistive term in z_i will play a role in it. However, an analytical form of R_o is currently unknown. Neglecting such resistance, substituting Eq. (12) into Eq. (16) suggests that at resonance when the reactance of z_t vanishes

$$\frac{r_i^2 \pi r_o^2/\mathrm{i}kV + \mathrm{i}kr_i r_o L}{r_o^2 + r_i r_o L \pi r_o^2/V} + \mathrm{i}kL_p = 0 \tag{17}$$

and the resonance frequency f_{res} becomes

$$f_{res} = \frac{c}{2\pi}\sqrt{\frac{\pi r_i^2}{L_n V}\left[\frac{r_i L}{r_o L_n}\left(1 + \frac{\pi r_o^2 L_p}{V}\right) + \frac{L_p}{L_n}\right]^{-1}}. \tag{18}$$

In reality, L_n is replaced by the effective length which is commonly taken to be $L_n + 16r_i/3\pi$ [12]. It can be directly deduced from Eq. (18) that f_{res} decreases as L_p increases no matter L_n is kept fixed or not. However, the accuracy of Eq. (18) depends on how large the resistance in z_i is. The weaker this resistance, the better the accuracy anticipated. Eq. (16) suggests that Eq. (18) will underestimate the resonance frequency if the resistance in z_i is significant.

The above conclusions may not apply to the case where the neck inlet is larger than its outlet, that is $r_i > r_o$. The resonance frequency will still be increased if one uses the frequency corresponding to the cavity inlet area of πr_o^2 as the reference, which is a trivial choice due to the manufacturing process. The change in total resistance z_i is not really certain. While R_o is unchanged, the tapering with $r_i > r_o$ amplifies the effect of R_o, but the radiation resistance at the neck inlet is reduced by a factor of r_o^2/r_i^2 [12]. However, this $r_i > r_o$ condition is not recommended as it tends to introduce additional flow separation at the neck inlet, which is exposed to fluid flow in practice.

3. Experimental set-up

The experiment was carried out using the Brüel & Kjær Type 4206 impedance tube assembly with the analyzer Brüel & Kjær 2144 and the software BZ5050 provided by Brüel & Kjær company. A full details on the Brüel & Kjær Type 4206 impedance tube assembly and the standard two-microphone method employed can be found in Ref. [16].

The Helmholtz resonator was made of Perspex and its cavity was cylindrical with a diameter of 94 mm. The design of the Helmholtz resonator enabled the built-in clamps of the impedance tube to fit the resonator tightly to the end of the impedance tube of diameter 100 mm, which was designed for frequency less than 1600 Hz. Fig. 2 shows the details of the Helmholtz resonator and the measurement assembly, except the personal computer on which BZ5050 was installed. Since the diameter of the impedance tube was fixed, the volume of the resonator was varied by altering the length of the resonator. This was achieved by moving the resonator rod, whose one end was fixed to the end-plate inside the resonator. The plates containing the resonator necks were fixed in position at the opened end of the resonator main body (cavity) by force fitting with thin plastic tapes on its circumference to prevent sound leakage at least within the frequency range concerned.

In the present paper, thickness of the plate containing the neck is 5.3 or 9 mm and the orifice diameter is fixed at 5 mm ($r_i = 2.5$ mm). Experiment with 2.5 mm total neck thickness has also been carried out, but the corresponding results are largely in-line with those discussed later and thus will not be included in this paper. The depth of the tapered section can also be varied and for a fixed L_n, the percentage of neck tapering is defined hereinafter as $L/L_n \times 100\%$. The slope m is kept at $\tan^{-1}(\frac{8}{9})$.

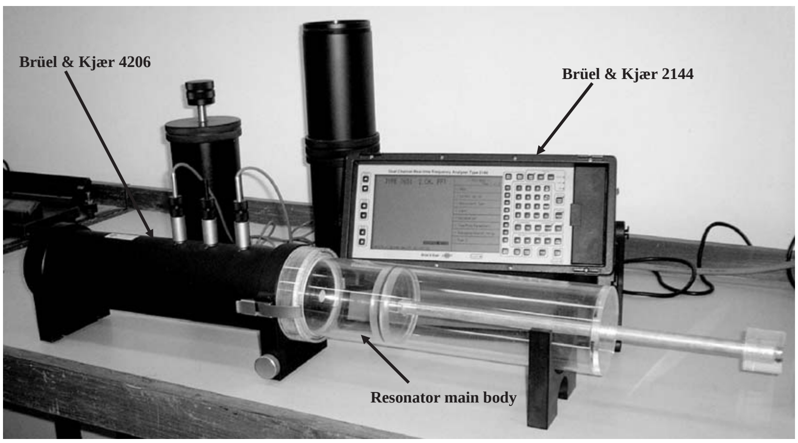
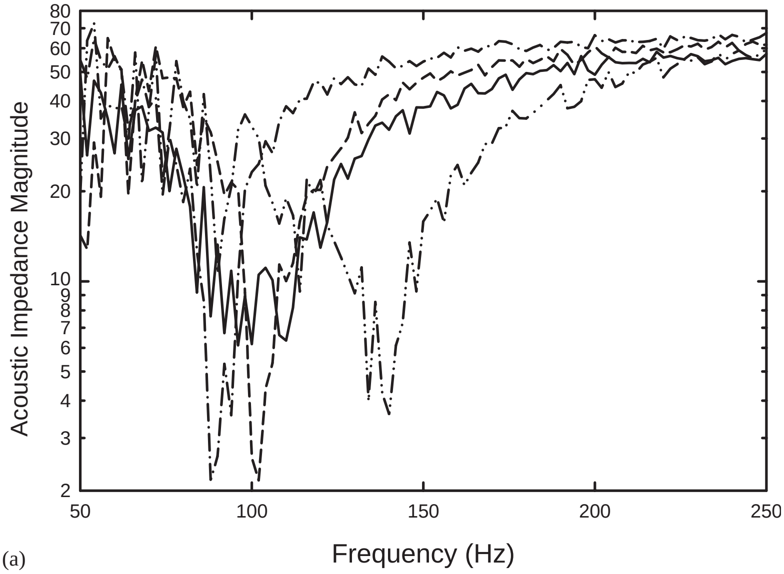


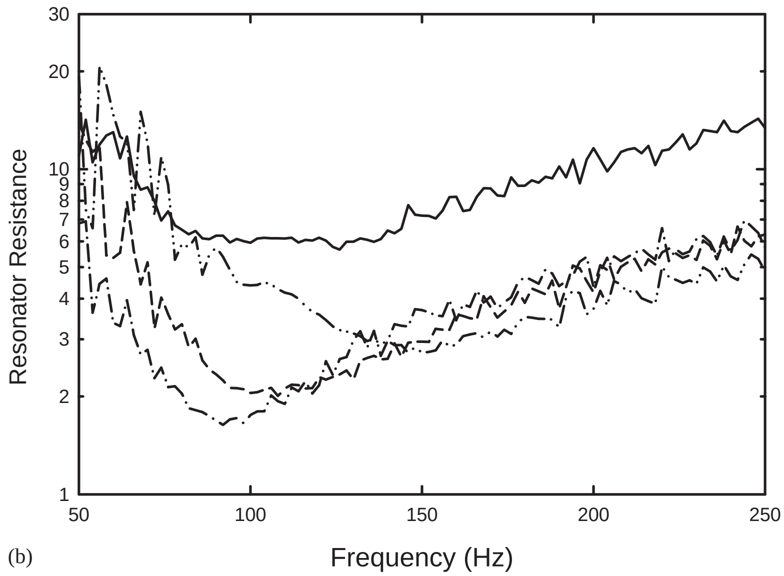
Fig. 2. The experimental test rig assembly.

4. Results and discussions

Fig. 3a shows some examples of the frequency variations of the nearly fully tapered neck Helmholtz resonator acoustic impedance magnitudes obtained in the present study, with a frequency resolution of 2 Hz, $L_n = 5.3$ mm and $L_p = 0.8$ mm ($\sim 85\%$ tapered). These impedance



(a)



(b)

Fig. 3. Frequency variations of acoustic properties of a tapered neck Helmholtz resonators. (a) Acoustic impedance magnitudes; (b) resistance. — ·· — : $L_c = 100$ mm, 85% tapered; – – – : $L_c = 160$ mm, 85% tapered; — · — : $L_c = 220$ mm, 85% tapered; —— : $L_c = 100$ mm, 0% tapered (reference). $L_n = 5.3$ mm.

magnitudes are normalized by that of air (ρc). All of these curves show a common feature of the resonator—impedance magnitude is the weakest near to the resonance frequency. All others are similar and thus not presented. One should note that the frequency of weakest impedance magnitude only corresponds to that of maximum sound absorption. The resonance frequency f_{res} is the frequency at which the reactance of the resonator impedance vanishes, though it is very close to that of highest sound absorption. An increase or decrease of the latter implies similar change of the former. It can be observed from Fig. 3b that the large impedance magnitudes away from the lowest values are due to the reactive actions of the resonators. In the present study, the measured impedance magnitudes never fall below that of the air, indicating that a decrease in z_i or z_t at the resonance frequency enhances the sound absorption performance of the resonator. It is evident from Fig. 3a that longer cavity length and thus larger cavity volume increases the sound absorption produced by the resonators.

Fig. 4 illustrates some typical frequency variations of sound absorption coefficients obtained in the present study with a cavity length L_c of 160 mm. The corresponding frequency resolution is again 2 Hz. Increases in the resonance frequency and the maximum sound absorption coefficient by tapering the neck are confirmed. The increase of the latter appears broadband. Fig. 4 also indicates that a small increase in the sound absorption coefficient as the tapered neck length L increases at fixed L_n. This is the outcome of a decrease in the acoustic impedance magnitude. It tends to agree with the theoretical deduction from Eq. (16) as all the measured acoustic impedances at the resonance frequencies f_{res} are greater than that of the air. Results presented in Fig. 4 also tend to suggest f_{res} increases as L_p is reduced (thus L increases).

The maximum sound absorption of the resonator is enhanced by introducing the neck tapering. The deeper the tapering, the higher this sound absorption will become (Fig. 5a). This appears

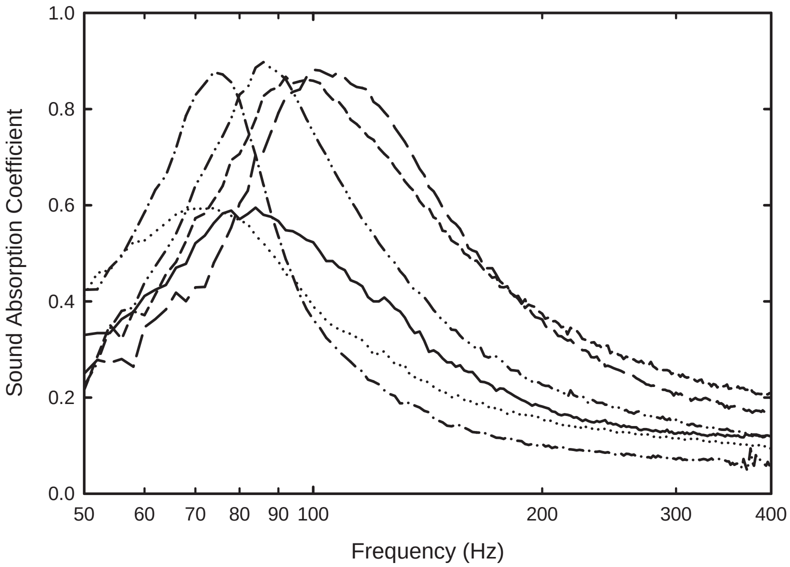


Fig. 4. Examples of frequency variations of sound absorption coefficients of Helmholtz resonators: —— : $L_n = 5.5$ mm, 0% tapered; – – – : $L_n = 5.5$ mm, 69% tapered; — — : $L_n = 5.5$ mm, 85% tapered; ⋯⋯ : $L_n = 9$ mm, 0% tapered; — · — : $L_n = 9$ mm, 49% tapered; — ·· — : $L_n = 9$ mm, 69% tapered. $L_c = 160$ mm.

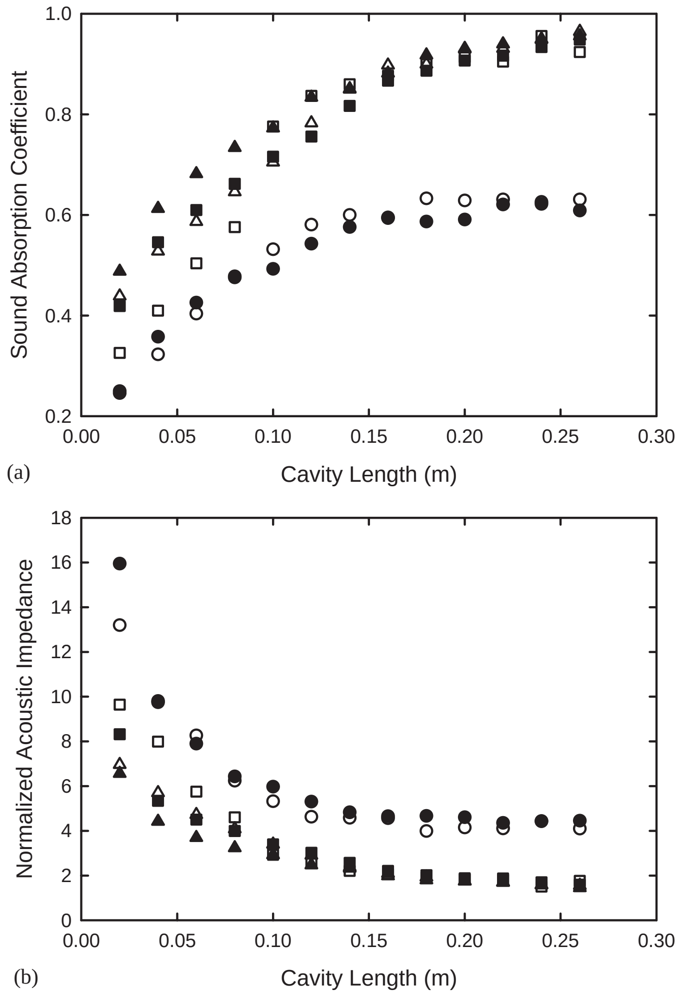


Fig. 5. Effect of neck tapering on acoustic performance of Helmholtz resonators. (a) Maximum sound absorption coefficients; (b) acoustic impedance magnitudes at resonance: $L_n = 5.3$ mm, ● : 0% tapered; ■ : 69% tapered; ▲ : 85% tapered. $L_n = 9$ mm, ◦ : 0% tapered; □ : 49% tapered; △ : 69% tapered.

consistent with the theoretical deduction though an exact formulation of such absorption cannot be given in the present study. One can observe that the absorption and thus the impedance of the resonators do not appear to depend on the neck thickness, especially under the untapered neck condition ($m = 0$), unless the cavity length is very small when compared to the cavity diameter. This is rather expected as the both the flow resistance and the radiation resistance are not very sensitive to the thickness of a straight neck. The sound absorption increases with cavity volume, but those of the untapered neck resonators flatten out at $L_c > 160$ mm. Fig. 5b shows the variation of acoustic impedance magnitude/resistance at the resonance frequency with cavity volume. It is obvious that this variation has a trend opposite to that of the sound absorption [12]. The resistance decreases with increasing cavity volume as expected. One can also observe that the

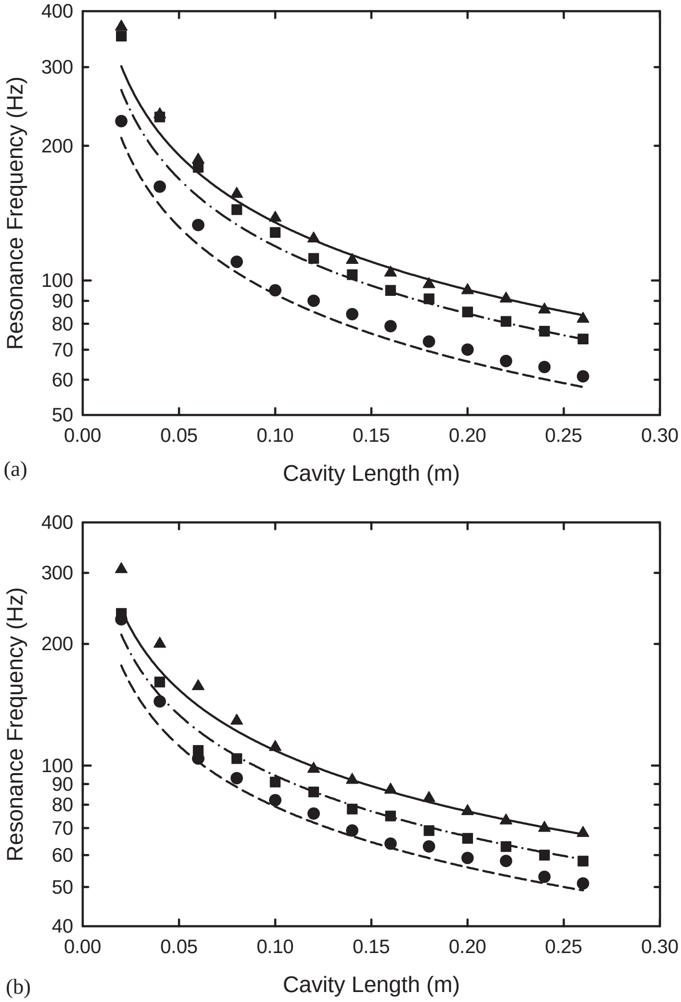


Fig. 6. Effect of neck tapering on resonance frequencies of Helmholtz resonators. (a) $L_n = 5.3$ mm; Theory, — — : 0% tapered; — · — : 69% tapered; —— : 85% tapered. Experiment, ● : 0% tapered; ■ : 69% tapered; ▲ : 85% tapered. (b) $L_n = 9$ mm; Theory, — — : 0% tapered; — · — : 49% tapered; —— : 69% tapered. Experiment, ● : 0% tapered; ■ : 49% tapered; ▲ : 69% tapered.

acoustic impedance magnitudes of the tapered neck resonators at large cavity volume are close to that of air. The tapering has reduced more than 60% of the impedance in the untapered condition. The corresponding flow resistance into the cavity is weak and thus better accuracy of Eq. (18) in the prediction of resonance frequency is expected.

The predicted resonator resonance frequencies in general agree with the experimental results, especially for the tapered neck resonators (Figs. 6a and b). Again better prediction can be achieved with deeper tapering length L, regardless the neck thickness L_n. One should note that the accuracy of resonance frequency predictions at small L_c is not going to be very satisfactory, as the length-to-diameter ratios of the cavity volumes at these cavity lengths are small. The effects of such cavity aspect ratio to untapered neck resonators are discussed by Dickey and Selamet [7]. However, such small aspect ratio is uncommon for conventional Helmholtz resonator.

It is also noticed from Fig. 6 that the theory depicted in Kinsler et al. [12] underestimates the resonance frequency for the untapered neck cases. The theory of Chanaud [17], which includes in-depth analysis of the length correction for the untapered neck, gives better prediction of the resonance frequency, though it still underestimates this frequency. For resonators of dimensions and aspect ratios similar to those involved in the present study, the theory of Chanaud [17] can result in about 4% better estimation on the resonance frequency in the untapered neck cases than the conventional theory shown in Kinsler et al. [12].

5. Conclusions

The sound absorption performance of a Helmholtz resonator with a tapered neck is investigated in the present study. A measurement test rig with a standard impedance measurement assembly was setup. The sound absorption coefficients and the acoustic impedance of the resonators up to 1600 Hz were measured. A simplified theory assuming plane wave propagation inside the tapered neck is derived to explain the experimental observation and to predict the resonance frequency of a tapered neck resonator. The effects of tapering length were also discussed.

Both the theoretical deduction and the experimental results indicate a reduction of the acoustic impedance, and thus an increase of the highest sound absorption coefficient attainable, of a Helmholtz resonator can be obtained by tapering the neck. The deeper the tapering length, the better the improvement of sound absorption will be. The sound absorption capacity of the resonator also increases with the cavity volume. Though a formula for the prediction of acoustic impedance is not yet developed, the present results suggest substantial increase in the sound absorption of the resonator can be achieved by tapering the neck to a depth more than 50% of the untapered neck length.

The resonance frequency increases with the tapering length as predicted by the present theory and confirmed by experiment. Such increase of resonance frequency decreases with increasing resonator cavity volume at fixed tapered section slope. The theory also shows that the resonance frequency increases with the tapered section slope at fixed cavity volume. The predicted resonance frequencies agree remarkably with the experimental observations. Again, the deeper the tapering length, the weaker the resistance in the neck-cavity function and the better the predictions obtained.

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